by fluid) bubbles rising toward the surface, we now have a more integrated structure consisting of bubbles and intervening fluid. The upward velocity of this chain is greater than the upward velocity of the individual bubbles and is determined by the buoyancy of the bubbles comprising the chain. The fluid entrained by the bubbles is displaced upward as such chains move toward the surface.

Calculations performed for another limiting case (surfacing of chains of bubbles in channels whose radius is comparable to a ($\lambda = 0.8$) showed that their upward velocity and the flow pattern are the same as in the surfacing of a single bubble if the distance between bubbles is greater than the diameter of the channel. This conclusion is fully in accord with the calculations in [3], where it was found that the velocity profile for the cross section of the tube smooths out at a distance of just 1.5 tube diameters from the bubble.

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STABILITY OF AN ADIABATIC CONTINUOUS CHEMICAL REACTOR

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Numerical studies (see [1-3], for example) have shown that there can be many steady-state regimes of operation of chemical reactors with distributed parameters. At the same time, as was demonstrated by N. N. Moiseev et al. in a postscript to [4], numerical methods cease to work for such systems in the neighborhood of bifurcation points – where the solution loses its uniqueness. Formidable obstacles are encountered in attempts to develop numerical methods of post-bifurcation analysis that make it possible to find all of the solutions emanating from bifurcation points. These obstacles are particularly great in the case of multi-dimensional problems or problems with many factors, such as in numerical studies of chemical reactors with distributed parameters.

In the present investigation, we use the theory in [4] to develop a method of analyzing the stability of steady-state solutions of a system of partial differential equations which describes the operation of a continuous chemical reactor with an adiabatic temperature change. The method is based on reduction of the number of dimensions of an infinite-dimensional problem through the use of projections of its solutions on an eigenfunction space and the Fredholm

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alternative. The method is used to find zeroth, bifurcative, and isolated (eliminating the bifurcation) solutions and to determine their stability. The analysis of the stability of solutions obtained in a space of the dimensionality R^2 is based on the Liapunov theorem and the Hopf hypothesis on the equivalence of strict loss of stability and a double bifurcation point [5].

It is assumed that the rate of heat release in the reactor is a continuous function $\varphi(c, T)$ of the temperature and concentrations of the reactants. It is further assumed that $\partial\varphi(c, T)/\partial T > 0$. This assumption is valid for any reactions characterized by an Arrhenius rate of heat release.

1. Formulation of the Problem. The mathematical description of the process which takes place in a continuous chemical reactor has the form [6]

$$\frac{\partial T(x, t)}{\partial t} = \varkappa \frac{\partial^2 T(x, t)}{\partial x^2} - w \frac{\partial T(x, t)}{\partial x} + \frac{Qz}{c_p} \varphi(c, T); \tag{1.1}$$

$$\frac{\partial c(x, t)}{\partial t} = D \frac{\partial^2 c(x, t)}{\partial x^2} - w \frac{\partial c(x, t)}{\partial x} - z\varphi(c, T), \tag{1.2}$$

where x is a coordinate; t is time; T is temperature; κ is diffusivity; c is concentration; Q is the heat of reaction referred to a unit mass; z is the preexponential multiplier; E is the activation energy; c_p is the heat capacity; R is the universal gas constant; w is the flow velocity; D is the diffusion coefficient; $\varphi(c, T)$ is a continuous function of the concentrations and temperature. Without loss of generality, we assume that a reaction of the Langmuir–Hinshelwood type with an Arrhenius rate of heat release takes place in the reactor:

$$\varphi(c, T) = \frac{k_1 c \exp(-E(RT)^{-1})}{(1 + k_2 c)^2} \tag{1.3}$$

(k_1 and k_2 are constants).

We use the following relations as the initial and boundary conditions

$$\partial T(0, t)/\partial x = -\alpha(T(0, t) - T_0),\ \partial c(0, t)/\partial x = -\alpha_c(c(0, t) - c_0), \tag{1.4}$$

$$\partial T(L, t)/\partial x = \partial c(L, t)/\partial x = 0;$$

$$T(x, 0) = T_0,\ c(x, 0) = c_0 \tag{1.5}$$

(α and α_c are constants and L is the length of the reactor). The condition of adiabaticity gives the below relationship between temperature and concentration:

$$T(x, t) = T_0 + Qc_p^{-1}(c_0 - c(x, t)). \tag{1.6}$$

After we change over to dimensionless parameters

$$\Theta = E(T - T_0)R^{-1}T_0^{-2},\quad \tau = tt_a^{-1},\quad \eta = xx_a^{-1},$$
$$u = wt_a x_a^{-1},\quad \delta = Lx_a^{-1},\quad \beta = RT_0E^{-1},\quad \alpha_1 = \alpha x_a$$
$$\left(t_a = c_p RT_0^2(EQz)^{-1}\exp(E(RT_0)^{-1}),\quad x_a = (\varkappa t_a)^{0.5}\right)$$

we introduce the operators

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial^2\Theta}{\partial\eta^2} - u\frac{\partial\Theta}{\partial\eta} + \sum_{n=1}^{\infty} b_n\Theta^n = F(\Theta, \mu); \tag{1.7}$$

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial^2\Theta}{\partial\eta^2} - u\frac{\partial\Theta}{\partial\eta} + b_1\Theta = \frac{\partial F(0, 0)}{\partial\Theta}\Theta + \mu\frac{\partial^2 F(0, 0)}{\partial\mu\,\partial\Theta}\Theta; \tag{1.8}$$

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial F(0, 0)}{\partial\Theta}\Theta + \mu\frac{\partial^2 F(0,0)}{\partial\mu\,\partial\Theta}\Theta + b_0 + \sum_{n\geqslant 2}^{\infty} b_n\Theta_n = G(\mu, \Theta, b_0), \tag{1.9}$$

where μ is a parameter from an interval containing zero;

$$b_n = \frac{1}{n!} \frac{\partial^n}{\partial \Theta^n} \varphi(\Theta) \bigg|_{\Theta=0};$$

$$\varphi(\Theta) = \frac{k_1 \left(c_0 - c_p (QE)^{-1} RT_0^2 \Theta\right) \exp\left(\Theta (1 + \beta\Theta)^{-1}\right)}{\left(1 + k_2 \left(c_0 - c_p (QE)^{-1} RT_0^2 \Theta\right)\right)^2}.$$

Conditions (1.4) and (1.5), written in dimensionless form, are valid for each of the operators (1.7)-(1.9).

Thus, nonlinear operator (1.9) is system (1.1), (1.2) written in dimensionless parameters with allowance for (1.6). Here, the nonlinear function $\varphi(\Theta)$ is represented in the form of a series in powers of Θ. Thus, the above-formulated problem reduces to determination of steady-state solutions of the given operator with conditions (1.4), (1.5) and subsequent analysis of their stability. Operators (1.7), (1.8) are obtained from (1.9) by successive elimination of the defect b_0 ($b_0 = 0$) and linearization.

2. Zero Solution. Analysis of the stability of the zero solution of system (1.1)-(1.6) reduces to the problem of determining the eigenvalues of operator (1.8) with conditions (1.4), (1.5). The spectrum of the operator (1.8) on the interval $(0, \delta)$ consists only of discrete eigenvalues $\sigma_n = b_1 - u^2/4 - \lambda_n^2$, where λ_n ($n = 1, 2, \ldots$) are positive roots of the equation

$$\operatorname{tg} \lambda\delta = -\frac{4\alpha_1\lambda}{4\lambda^2 + u^2 + 2u\alpha_1}. \tag{2.1}$$

The zero solution is stable if the maximum value $\sigma_{max} = \sigma_1 < 0$. The quantity σ_1 will henceforth be identified with the parameter μ, and the boundary of stability of the zero solution will be determined from the equation

$$\mu = b_1 - u^2/4 - \lambda_1^2 = 0. \tag{2.2}$$

Simultaneous solution of (2.1), (2.2) gives the relation for the critical value of δ:

$$\delta = \frac{2}{\sqrt{4b_1 - u^2}} \left[\operatorname{arctg}\left(-\frac{\alpha_1\sqrt{4b_1 - u^2}}{2b_1 + u\alpha_1}\right) + a\pi\right].$$

Here, $a = 0$ at $\alpha_1\sqrt{4b_1 - u^2}(2b_1 + u\alpha_1)^{-1} < 0$; $a = 1$ at $\alpha_1\sqrt{4b_1 - u^2}\,(2b_1 + u\alpha_1)^{-1} \geq 0$.

All of the eigenvalues σ_n of operator (1.8) are double eigenvalues, and each corresponds to two eigenvectors:

$$y_{1n} = \cos\lambda_n\eta \exp(0{,}5u\eta),\quad y_{2n} = -(0{,}5u + \alpha_1)\sin\lambda_n\eta \exp(0{,}5u\eta).$$

The vectors y_{1i}, y_{2j} ($i, j = 1, 2, \ldots$) are independent and form a complete system on $(0, \delta)$. Thus, (1.7) and (1.9) can be regarded as certain evolutionary problems in the space R^∞ formed by these vectors.

3. Bifurcative Solution. To solve (1.7) with conditions (1.4), (1.5), we introduce transforms which reduce the system of vectors y_{ij} ($i = 1, 2$, $j = 1, 2, \ldots$) to a biorthogonal system of vectors $\bar{y}_{ij}$ with the weight $\exp(-u\eta)$. Thus, the eigenvalue σ_1 will correspond to the vectors $\bar{y}_{11} = y_{11}$, $\bar{y}_{21} = y_{21} - \langle y_{21}, \bar{y}_{11}\exp(-u\eta)\rangle \|y_{11}\|^{-1}\bar{y}_{11}$, where $\langle y_{ij}, y_{nm}\rangle$ is the scalar product of the vectors y_{ij}, y_{nm}, $\|\bar{y}_{ij}\| = \langle \bar{y}_{ij}, \bar{y}_{ij}\exp(-u\eta)\rangle$.

The vectors $\bar{y}_{ij}$ ($i = 1, 2$, $j = 1, 2, \ldots$) form a Hilbert space H with the scalar product

$$\langle(\bar{y}_{1i}, \bar{y}_{2j}), (\bar{y}_{1n}, \bar{y}_{2m})\rangle = \langle \bar{y}_{1i}, \bar{y}_{1n}^*\rangle + \langle \bar{y}_{2j}, \bar{y}_{2m}^*\rangle$$

($\bar{y}_{1j}^*$, $\bar{y}_{2j}^*$ are the vectors conjugate to $\bar{y}_{1j}$, $\bar{y}_{2j}$). Since the vectors $\bar{y}_{ij}$ are orthogonal on $(0, \delta)$ with the weight $\exp(-u\eta)$, we should take the following as the vectors conjugate to

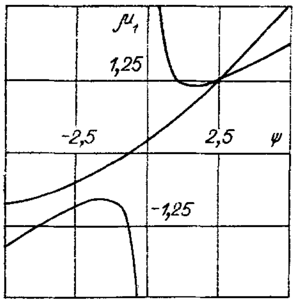

Fig. 1

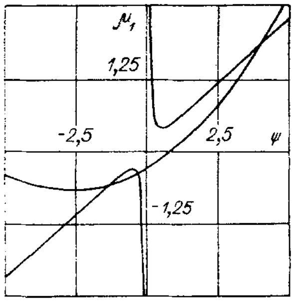

Fig. 2

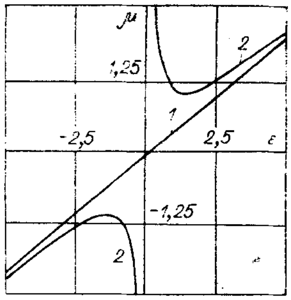

Fig. 3

$\bar{y}_{11}$, $\bar{y}_{21}$

$$\bar{y}_{i1}^{*} = \bar{y}_{i1} \exp(-u\eta) \left(\| \bar{y}_{11} \| + \| \bar{y}_{21} \| \right)^{-1} \quad (i = 1, 2).$$

By virtue of the orthogonality of the vectors $\bar{y}_{ij}$, the solution $\Theta = \Theta(\mu)$ of operator (1.7) can always be broken down into a part belonging to the two-dimensional null-space of operator (1.8) and a part which is orthogonal to $\bar{y}_{11}^{*}$, $\bar{y}_{21}^{*}$.

We seek the solution of (1.7) in the form of series

$$\Theta = \sum_{n=1}^{\infty} \Theta_n \frac{\varepsilon^n}{n!}, \quad \mu = \sum_{n=1}^{\infty} \mu_n \frac{\varepsilon^n}{n!} \tag{3.1}$$

($\varepsilon = \langle (\Theta, \Theta), (\bar{y}_{11}, \bar{y}_{21}) \rangle$ is the amplitude).

Substitution of (3.1) into (1.7) and identification of the terms with the powers of ε leads us to equations relative to ε, ε^2:

$$\partial F(0, 0)/\partial\Theta\Theta_1 = 0; \tag{3.2}$$

$$\partial F(0, 0)/\partial\Theta\Theta_2 + 2\mu_1 \partial^2 F(0, 0)/\partial\Theta\, \partial\mu \Theta_1 + \partial^2 F(0, 0)/\partial\Theta^2 \Theta_1^2 = 0. \tag{3.3}$$

It follows directly from (3.2) that any linear combination $\Theta_1 = \bar{y}_{11} + \psi \bar{y}_{21}$ can be a solution (ψ is a parameter of the problem which is subject to determination).

Equation (3.3) can be solved only when — in accordance with the Fredholm alternative for $k = 1, 2$ — the conditions $\langle \partial F(0, 0)/\partial\Theta\Theta_2, \bar{y}_{k1}^{*} \rangle = 0$ are satisfied and, thus,

$$2\mu_1 \langle \partial^2 F(0, 0)/\partial\Theta\, \partial\mu\, \Theta_1, \bar{y}_{k1}^{*} \rangle + \langle \partial^2 F(0, 0)/\partial\Theta^2 \Theta_1^2, \bar{y}_{k1}^{*} \rangle = 0. \tag{3.4}$$

Insertion of the expressions for Θ_1, $\bar{y}_{k1}^{*}$ $(k = 1, 2)$ into (3.4) leads to two conic equations in the (μ_1, ψ) plane:

$$g_1(\mu_1, \psi) = c_{11}\psi^2 + c_{12}\psi + c_{13}\mu_1\psi + c_{14}\mu_1 + c_{15} = 0; \tag{3.5}$$

$$g_2(\mu_1, \psi) = c_{21}\psi^2 + c_{22}\psi + c_{23}\mu_1\psi + c_{24}\mu_1 + c_{25} = 0. \tag{3.6}$$

Here,

$$c_{11} = 0{,}5 \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{21}^2, \bar{y}_{11}^{*} \rangle, \quad c_{12} = \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{11}\bar{y}_{21}, \bar{y}_{11}^{*} \rangle,$$
$$c_{13} = \langle \partial^2 F(0, 0)/\partial\Theta\, \partial\mu \bar{y}_{21}, \bar{y}_{11}^{*} \rangle, \quad c_{14} = \langle \partial^2 F(0, 0)/\partial\Theta\, \partial\mu \bar{y}_{11}, \bar{y}_{11}^{*} \rangle,$$
$$c_{15} = 0{,}5 \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{11}^2, \bar{y}_{11}^{*} \rangle, \quad c_{21} = 0{,}5 \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{21}^2, \bar{y}_{21}^{*} \rangle,$$
$$c_{22} = \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{11}\bar{y}_{21}, \bar{y}_{21}^{*} \rangle, \quad c_{23} = \langle \partial^2 F(0, 0)/\partial\Theta\, \partial\mu \bar{y}_{21}, \bar{y}_{21}^{*} \rangle,$$
$$c_{24} = \langle \partial^2 F(0, 0)/\partial\Theta\, \partial\mu \bar{y}_{11}, \bar{y}_{21}^{*} \rangle, \quad c_{25} = 0{,}5 \langle \partial^2 F(0, 0)/\partial\Theta^2 \bar{y}_{11}^2, \bar{y}_{21}^{*} \rangle.$$

Due to the orthogonality of the vectors $\bar{y}_{11}$, $\bar{y}_{21}$, Eq. (3.5) always describes a parabola and Eq. (3.6) always describes a hyperbola. The points of intersection of curves (3.5), (3.6) $(\mu_1^{(n)}, \psi^{(n)})$

are solutions of (3.3). There may be one, two, or three points of intersection in the region of real values of μ_1, ψ corresponding to the steady-state solutions. Thus, when $c_0\rho^{-1} = 1$, $k_1 = 1$, $k_2 = 0$, $RT_0c_p(EQ)^{-1} = 0.1$, $\beta = 0$ (which gives $b_1 = 0.9$, $b_2 = 0.4$), $\delta = 2.50$, $\alpha_1 = 0.1$, $u = 1$, curves (3.5), (3.6) intersect at one point $(\mu_1^{(1)}, \psi^{(1)}) = (1.24, 2.40)$ (Fig. 1), while then $\delta = 2.49$, $\alpha_1 = 1$, $u = 0.1$, $b_1 = 0.9$, $b_2 = 0.4$ they intersect at three points $(\mu_1^{(1)}, \psi^{(1)}) = (1.88, 4.05)$, $(\mu_1^{(2)}, \psi^{(2)}) = (-0.54, -1.20)$, $(\mu_1^{(3)}, \psi^{(3)}) = (-0.35, -0.32)$ (Fig. 2). Since c_0, ρ, k_1, k_2, R, T_0, c_p, E, Q, β enter into (1.4), (1.5), (1.7)-(1.9) only through the coefficients b_i ($i = 1, 2, \ldots$), we will henceforth present only the values of these coefficients.

Thus, in the plane (μ, ε) the bifurcative solutions form the family of curves $\mu = \mu_1^{(n)}\varepsilon$. Figure 3 (straight line 1) shows the unique solution $\mu = \mu_1^{(1)}\varepsilon$ obtained when $\delta = \pi/2$, $u = 1$, $\alpha_1 = -0.5$, $b_1 = 0.9$, $b_2 = 0.4$. Here, it would be appropriate to remark once more that the possibility of the existence of from one to three solutions to the given problem was proven by numerical methods for $\varphi(c, T) \sim c^n$ in [1] and for autocatalytic reactions of the Langmuir–Hinshelwood type in [3].

To analyze the stability of solutions (1.7) at the points $(\mu_1^{(n)}, \psi^{(n)})$, it is necessary to represent the relations $g_i(\mu_1, \psi)$ ($i = 1, 2$) in the form of functions of the parameter μ. Combining (3.1), (3.5), and (3.6) and using the normalization condition $\varepsilon = 1$, we obtain

$$\bar{g}_i(\mu) = \mu^2\left(c_{i1}\frac{\psi^2}{\mu_1^2} + c_{i2}\frac{\psi}{\mu_1^2} + c_{i3}\frac{\psi}{\mu_1} + c_{i4}\frac{1}{\mu_1} + c_{i5}\frac{1}{\mu_1^2}\right) \quad (i = 1, 2). \tag{3.7}$$

System (3.7) can be regarded as two transformed ordinary differential equations. Thus, the stability of their solutions can be analyzed by a method based on the stability theorem corresponding to the first Liapunov approximation (see [7] as an example). In accordance with this method, solution (1.7) with conditions (1.4), (1.5) at the point $(\mu_1^{(n)}, \psi^{(n)})$ is stable if the eigenvalues $s_1^{(n)}$, $s_2^{(n)}$ of the Jacobian matrix

$$I = \begin{vmatrix} \partial\bar{g}_1(\mu)/\partial(\mu_1^{-1}) & \partial\bar{g}_1(\mu)/\partial(\psi\mu_1^{-1}) \\ \partial\bar{g}_2(\mu)/\partial(\mu_1^{-1}) & \partial\bar{g}_2(\mu)/\partial(\psi\mu_1^{-1}) \end{vmatrix}$$

are negative. Considering that at each point we have $\det I = \mu^2 \det I(\mu_1^{(n)}, \psi^{(n)}) + 0|\mu^3|$, we can write the stability condition in the form

$$\max(\mu s_1^{(n)}, \mu s_2^{(n)}) < 0. \tag{3.8}$$

Condition (3.8) is valid only when curves (3.5) and (3.6) intersect transversely at the given point. The condition of transversality at the n-th point of intersection $\det I_0(\mu_1^{(n)}, \psi^{(n)}) \neq 0$.

where

$$I_0 = \begin{vmatrix} \partial g_1(\mu_1, \psi)/\partial\mu_1 & \partial g_1(\mu_1, \psi)/\partial\psi \\ \partial g_2(\mu_1, \psi)/\partial\mu_1 & \partial g_2(\mu_1, \psi)/\partial\psi \end{vmatrix},$$

is evidence of bifurcation of the solution at this point. The bifurcation points form the subspace $\Omega_1(u, \delta, \alpha_1, b_0, b_1, b_2)$ of the phase space of states of the reactor $\Omega = \Omega(u, \delta, \alpha_1, b_0, b_1, b_2)$ (the parameters in the parentheses are regarded as bifurcative parameters).

If at the point of intersection of the conic sections there is a common tangent such that $\det I_0 = 0$, then system (3.5), (3.6) will have one solution on one side of this point and three solutions on the other side. If the solution of (3.5), (3.6) is a point for which $\det I_0 = 0$, then there is yet one more solution with $\det I_0 \neq 0$.

The transversality condition is satisfied for all points of intersection of the curves shown in Figs. 1 and 2 (as is evident from the figures themselves). An analysis of stability performed using (3.8) showed that for $(\mu_1^{(1)}, \psi^{(1)})$ (Fig. 1) $(s_1^{(1)}, s_2^{(1)}) = \mu(0.40, -0.72)$ and the solution is unstable on both sides of the critical point $\mu = 0$, while for

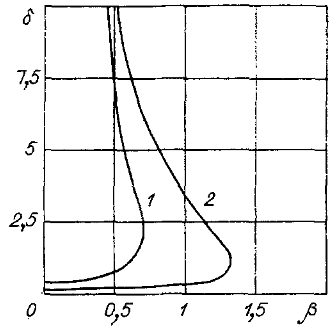

Fig. 4

$(\mu_1^{(1)}, \psi^{(1)})$, $(\mu_1^{(2)}, \psi^{(2)})$, $(\mu_1^{(3)}, \psi^{(3)})$ (Fig. 2) the eigenvalues of the matrix I are as follows: $(s_1^{(1)}, s_2^{(1)}) = \mu(1.26, -3.28)$, $(s_1^{(2)}, s_2^{(2)}) = \mu(0.87, 0.27)$, $(s_1^{(3)}, s_2^{(3)}) = \mu(0.38, -0.34)$. It follows from this that the solutions at the first and third points of intersection are unstable for any μ, but the solution at the second point is stable at $\mu < 0$ and unstable at $\mu > 0$.

Both solutions are unstable if $\det I < 0$ and are stable on one side of the point $\mu = 0$ if $\det I > 0$. It must be noted in connection with this that two successive points of intersection on one arc of the conic sections (3.5), (3.6) [such as $(\mu_1^{(2)}, \psi^{(2)})$, $(\mu_1^{(3)}, \psi^{(3)})$ in Fig. 2] have different signs for $\det I$. This means that one solution is stable for any μ, while the other is stable on one side of the point $\mu = 0$.

The equation $(\mu s_1^{(n)}, \mu s_2^{(n)}) = 0$ represents a hyperplane which divides Ω_1 into regions of stable and unstable bifurcative solutions. Thus, if we put $u = 1$, $\alpha_1 = -0.5$, $b_1 = 0.9$, $b_2 = 0.4 - \beta$ and reduce the subspace Ω_1 to a two-dimensional subspace $\Omega_1 = \Omega_1(1; \delta; -0.5; 0; 0.9; 0.4 - \beta)$, then the boundary delimiting the stable region above and below in the plane (δ, β) will have the form shown in Fig. 4 (curve 1).

4. Isolated Solutions. The solution $\Theta = 0$ of the equation $G(\mu, \Theta, 0) = 0$ becomes unstable with the passage of μ through zero. In accordance with the Hopf theorem [5], it follows from this that the point $(\mu, \Theta) = (0, 0)$ is a double point. To determine the stability of solution (1.9) with allowance for the defect $b_0 \neq 0$ which eliminates the bifurcation, we need to put $b_0 = \Delta(\mu, \varepsilon)$. Then it follows from the condition $\langle \partial G(0, 0, 0)/\partial\Delta, \bar{y}_{k1}^* \rangle$ $(k = 1, 2)$ that the steady-state solutions $G(\mu, \Theta, 0) = 0$, branching at the double point $(\mu, \Theta) = (0, 0)$ when $\Delta = 0$, become isolated solutions which eliminate the bifurcation when $\Delta \neq 0$.

Double differentiation of $G(\mu, \Theta, \Delta)$ with respect to μ, ε at the point $(\mu, \varepsilon) = (0, 0)$ leads to the system of equations

$$\frac{\partial G(0,0,0)}{\partial\Theta}\frac{\partial^2\Theta}{\partial\varepsilon^2} + \frac{\partial^2 G(0,0,0)}{\partial\Theta^2}\Theta_1^2 + \frac{\partial G(0,0,0)}{\partial\Delta}\frac{\partial^2\Delta}{\partial\varepsilon^2} = 0; \tag{4.1}$$

$$\frac{\partial G(0,0,0)}{\partial\Theta}\frac{\partial^2\Theta}{\partial\mu\,\partial\varepsilon} + \frac{\partial^2 G(0,0,0)}{\partial\mu\,\partial\Theta}\Theta_1 + \frac{\partial G(0,0,0)}{\partial\Delta}\frac{\partial^2\Delta}{\partial\mu\,\partial\varepsilon} = 0, \tag{4.2}$$

which can be solved only when the relations $\langle \partial^2\Theta/\partial\varepsilon^2, \bar{y}_{k1}^* \rangle = \langle \partial^2\Theta/\partial\mu\partial\varepsilon, \bar{y}_{k1}^* \rangle = 0$ are satisfied for $k = 1, 2$. The latter relations, together with (4.1), (4.2), make it possible to determine the first two nontrivial terms in the expansion of $\Delta(\mu, \varepsilon)$ in powers of μ, ε:

$$\Delta(\mu, \varepsilon) = -\frac{1}{2}\left[\frac{\langle \partial^2 G(0,0,0)/\partial\Theta^2\Theta_1^2, \bar{y}_{k1}^* \rangle}{\langle \partial G(0,0,0)/\partial\Delta, \bar{y}_{k1}^* \rangle}\varepsilon^2 + 2\frac{\langle \partial^2 G(0,0,0)/\partial\Theta\,\partial\mu\Theta_1, \bar{y}_{k1}^* \rangle}{\langle \partial G(0,0,0)/\partial\Delta, \bar{y}_{k1}^* \rangle}\varepsilon\mu\right] \quad (k = 1, 2). \tag{4.3}$$

Curves (4.3) represent isolated solutions which eliminate the bifurcation. Figure 3 (curves 2) show such solutions obtained with $\delta = \pi/2$, $u = 1$, $\alpha_1 = -0.5$, $b_0 = 1$, $b_1 = 0.9$, $b_2 = 0.4$.

Substitution of the expressions for Θ_1, $\bar{y}_{k1}^*$, $k = 1, 2$ into (4.3) leads to two equations for conic sections:

$$g_1(\mu_1, \psi) + b_0\langle \partial G(0,0,0)/\partial\Delta, \bar{y}_{k1}^* \rangle = 0; \tag{4.4}$$

$$g_2(\mu_1, \psi) + b_0 \langle \partial G(0, 0, 0)/\partial\Delta, \bar{y}_{k1}^* \rangle = 0, \qquad (4.5)$$

where $g_i(\mu_1, \psi)$ ($i = 1, 2$) is found from (3.5), (3.6).

As (3.5) and (3.6), Eqs. (4.4), (4.5) can have one, two, or three solutions. However, the number of solutions of systems (4.4), (4.5) and (3.5), (3.6) does not have to coincide. At $\delta = 2.49$, $\alpha_1 = 1$, $u = 0.1$, $b_0 = 1$, $b_1 = 0.9$, $b_2 = 0.4$, curves (4.4), (4.5) intersect at three points $(\mu_1^{(1)}, \psi^{(1)}) = (2.98, 6.19)$, $(\mu_1^{(2)}, \psi^{(2)}) = (-1.43; -2.41)$, $(\mu_1^{(3)}, \psi^{(3)}) = (-1.31; -1.23)$.

Equations (4.4), (4.5) are similar in structure to (3.5), (3.6). Thus, the stability of solution (1.9) at the points of intersection of curves (4.4), (4.5) is studied in the same manner as was done in Part 3 for the bifurcative solution. The results of calculations of the above solutions of (4.4), (4.5) gave $(s_1^{(1)}, s_2^{(1)}) = \mu(3.55; -5.47)$, $(s_1^{(2)}, s_2^{(2)}) = \mu(1.68; 0.86)$, $(s_1^{(3)}, s_2^{(3)}) = \mu(1.39; -0.79)$, from which it follows that the solutions of (1.9) are unstable at the points $(\mu_1^{(1)}, \psi^{(1)})$, $(\mu_1^{(3)}, \psi^{(3)})$ on both sides of the point $\mu = 0$, while they are stable at the point $(\mu_1^{(2)}, \psi^{(2)})$ when $\mu < 0$.

Figure 4 (curve 2) shows the boundary between the regions of stability and instability of the solution which eliminates bifurcation in the two-dimensional subspace $\Omega_1 = \Omega_1(1; \delta; -0.5; 1; 0.9; 0.4 - \beta)$.

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